

# Distributed Interference Alignment with Low Overhead

YanJun Ma, Jiandong Li, *Senior Member, IEEE*, and Rui Chen *Student Member, IEEE*

**Abstract**—Based on closed-form interference alignment (IA) solutions, a low overhead distributed interference alignment (LOIA) scheme is proposed in this letter for the  $K$ -user single-input single-output (SISO) interference channel, and extension to multiple antenna scenario is also considered. Compared with the iterative interference alignment (IIA) algorithm proposed by Gomadam *et al.*, the overhead of our LOIA scheme is greatly reduced. Simulation results show that the IIA algorithm is strictly suboptimal compared with our LOIA algorithm in the overhead-limited scenario.

**Index Terms**—Interference channel, interference alignment, channel state information, iterative algorithm.

## I. INTRODUCTION

INTERFERENCE alignment has attracted much attention in recent years, which was introduced by Maddah-Ali *et al.* [1]-[2] for the multiple-input multiple-output (MIMO) X channel and subsequently by Cadambe and Jafar [3] in the context of the  $K$ -user interference channel (IC). The basic idea of IA is to align multiple interfering signals at each receiver in order to reduce the effective interference. It was shown in [3] that up to  $K/2$  total degrees-of-freedom (DoF) is achievable.

While the huge potential benefits can be achieved via IA, several challenges must be overcome before these benefits translated into practice. One key issue is the assumption of global channel state information (GCSI) at each node which is assumed in most papers. While a node may acquire CSI for its own channels, it is much harder to learn the channels between other pairs of nodes with which this node is not directly associated. To eliminate the GCSI assumption, an iterative interference alignment (IIA) algorithm was proposed in [4] based on channel reciprocity to align interference in a distributed fashion. However, the overhead induced by long iterations can overwhelm the gain achieved by IA [5].

In this letter, based on the closed-form IA solutions in [3], a low overhead distributed interference alignment (LOIA) scheme is proposed for the  $K$ -user single-input single-output (SISO) IC, and extension to multiple antenna scenario is also considered. The overhead of our LOIA scheme is greatly reduced compared with the IIA algorithm. Simulation results

show that the IIA algorithm is strictly suboptimal compared with our LOIA algorithm in the overhead-limited scenario.

We have noticed that concurrent work was developed in [9] based on the close-form solutions in [6]. However, their scheme is specially designed for FDD scenario where much overhead and delay are incurred while our scheme is based on the channel reciprocity in TDD scenario. It is pointed out in [5], [10] that IA is impractical when applied to many users in the IC simultaneously. So, we are more interested in the 3-user IC.

## II. SYSTEM MODEL AND PRELIMINARIES

Consider the  $K$ -user time-varying or frequency-selective IC consisting of  $K$  transmitter - receiver pairs. Each node is equipped with one antenna (multiple antenna scenario is considered later). The received signal at receiver  $k$  is given by

$$Y^{[k]} = \sum_{j=1}^K \mathbf{H}^{[kj]} X^{[j]} + Z^{[k]},$$

where  $X^{[j]}$  is an  $M \times 1$  vector that represents the transmitted signal of user  $j$  and  $M$  denotes the number of symbol extension of the channel in time or frequency.  $Z^{[k]}$  is a zero mean additive white Gaussian noise.  $\mathbf{H}^{[kj]}$  is the diagonal channel matrix between transmitter  $j$  and receiver  $k$ . Let  $d^{[k]}$  be the independent data streams sent from transmitter  $k$  to receiver  $k$ . Transmitter  $k$  sends a signal vector  $S^{[k]}$  to its intended receiver  $k$  by using a precoding matrix  $\mathbf{V}^{[k]}$ , i.e.,

$$X^{[k]} = \mathbf{V}^{[k]} S^{[k]}.$$

Receiver  $k$  estimates the transmitted data vector  $S^{[k]}$  by using a receive beamforming matrix  $\mathbf{U}^{[k]}$ , i.e.,

$$\tilde{S}^{[k]} = (\mathbf{U}^{[k]})^\dagger Y^{[k]},$$

where  $(\cdot)^\dagger$  stands for conjugate transpose.

### A. A Brief Review of IIA Algorithm

The IIA algorithm was proposed in [4] by utilizing the reciprocity of wireless channel. Fig. 1 shows the relationship between the number of iterations and the interference leakage power per user. We can see that long iterations are needed before data transmission began. The overhead induced by the iterations can overwhelm the gain achieved by IA [5].

## III. PROPOSED LOIA ALGORITHM

In this section, a low overhead distributed interference alignment (LOIA) scheme is proposed based on the close-form solutions in [3].

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The authors are with the State Key Laboratory of Integrated Service Networks, Xidian University, Xi'an 710071, China (e-mail: {yjm, jdli}@mail.xidian.edu.cn), rchenxidian@gmail.com.

### A. LOIA Algorithm for the $K$ -User SISO IC

We describe the LOIA algorithm for the 3-user SISO IC in detail. Extension to the  $K$ -user SISO IC can be realized based on Appendix III in [3]. For a limited space, we will not repeat it here. We show that  $\{d^{[1]}, d^{[2]}, d^{[3]}\} = \{n+1, n, n\}$  data streams can be sent simultaneously over a  $2n+1$  symbol extension in a distributed fashion. The process is divided into 3 phases and is shown in Fig. 2.

**Phase I:** Channel estimation phase.

**Phase II:** LOIA algorithm phase.

1) At RX1, RX2, and RX3, let

$$\mathbf{P}_1 = \mathbf{H}^{[12]}(\mathbf{H}^{[13]})^{-1},$$

$$\mathbf{P}_2 = \mathbf{H}^{[23]}(\mathbf{H}^{[21]})^{-1},$$

$$\mathbf{P}_3 = \mathbf{H}^{[31]}(\mathbf{H}^{[32]})^{-1},$$

respectively.

2) RXs send some training sequences along the vectors in  $\mathbf{P}_1$ ,  $\mathbf{P}_2$ , and  $\mathbf{P}_3$  respectively. Then  $\mathbf{P}_1$ ,  $\mathbf{P}_2$ , and  $\mathbf{P}_3$  can be estimated in all TXs.

3) Let  $\mathbf{w} = [1, 1, \dots, 1]^T$  be a  $(2n+1) \times 1$  column vector, which is a predefined vector and is known to all TXs. All TXs calculate

$$\mathbf{T} = \mathbf{P}_1 \mathbf{P}_2 \mathbf{P}_3.$$

At TX1, let

$$\mathbf{V}^{[1]} = [\mathbf{w} \quad \mathbf{T}\mathbf{w} \quad \mathbf{T}^2\mathbf{w} \quad \dots \quad \mathbf{T}^n\mathbf{w}].$$

At TX2, calculate  $\mathbf{C} = [\mathbf{w} \quad \mathbf{T}\mathbf{w} \quad \mathbf{T}^2\mathbf{w} \quad \dots \quad \mathbf{T}^{n-1}\mathbf{w}]$ , and let

$$\mathbf{V}^{[2]} = \mathbf{P}_3 \mathbf{C}.$$

At TX3, calculate  $\mathbf{B} = [\mathbf{T}\mathbf{w} \quad \mathbf{T}^2\mathbf{w} \quad \dots \quad \mathbf{T}^n\mathbf{w}]$ , and let

$$\mathbf{V}^{[3]} = \mathbf{P}_2^{-1} \mathbf{B}.$$

Then IA is achieved at all RXs, as  $\mathbf{V}^{[1]}$ ,  $\mathbf{B}$ ,  $\mathbf{C}$ , and  $\mathbf{T}$  satisfy

$$\mathbf{B} = \mathbf{T}\mathbf{C}, \quad (1)$$

$$\mathbf{B} \prec \mathbf{V}^{[1]}, \quad (2)$$

$$\mathbf{C} \prec \mathbf{V}^{[1]}, \quad (3)$$

where  $\mathbf{P} \prec \mathbf{Q}$ , means that the set of column vectors of matrix  $\mathbf{P}$  is a subset of the set of column vectors of matrix  $\mathbf{Q}$ . It is shown in [3] that (1)-(3) are equivalent to the IA conditions (4)-(6).

$$\mathbf{H}^{[12]}\mathbf{V}^{[2]} = \mathbf{H}^{[13]}\mathbf{V}^{[3]}, \quad (4)$$

$$\mathbf{H}^{[23]}\mathbf{V}^{[3]} \prec \mathbf{H}^{[21]}\mathbf{V}^{[1]}, \quad (5)$$

$$\mathbf{H}^{[32]}\mathbf{V}^{[2]} \prec \mathbf{H}^{[31]}\mathbf{V}^{[1]}. \quad (6)$$

4) TXs send some training sequences along vectors in  $\mathbf{V}^{[j]}$ ,  $j = 1, 2, 3$ . RXs estimate  $\mathbf{V}^{[j]}$  and let

$$\mathbf{U}^{[1]} = \text{null}(\mathbf{H}^{[12]}\mathbf{V}^{[2]}),$$

$$\mathbf{U}^{[2]} = \text{null}(\mathbf{H}^{[21]}\mathbf{V}^{[1]}),$$

$$\mathbf{U}^{[3]} = \text{null}(\mathbf{H}^{[31]}\mathbf{V}^{[1]}),$$

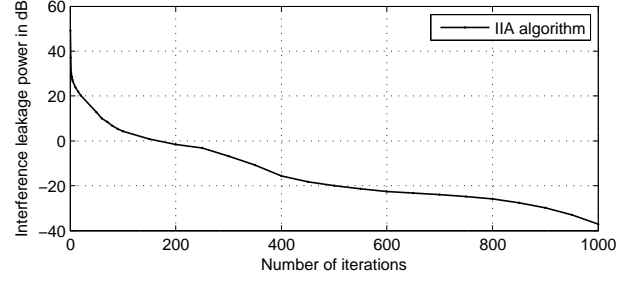


Fig. 1. Interference leakage power per user of the IIA algorithm (algorithm 1 in [4]) in the 3-user MIMO IC where each node is equipped with 2 antennas and one stream is transmitted per user-pair. The transmit power of each node is 40dB while the noise power is normalized to 0dB.

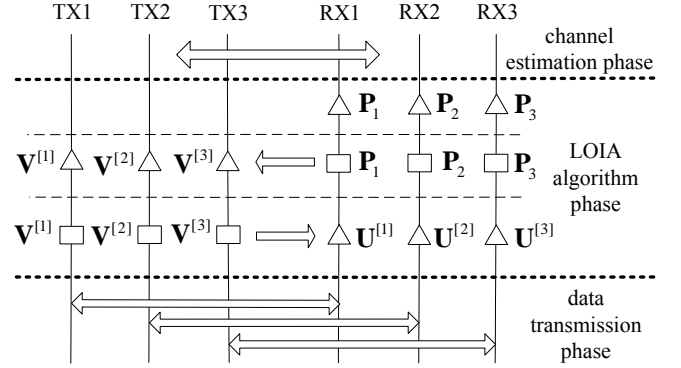


Fig. 2. The process of the LOIA algorithm for the 3-user IC, where the triangle marks mean that some calculations are performed on this node and the quadrangle marks mean that some training sequences are sent from this node along some vectors.

at RX1, RX2, and RX3 respectively, where  $\text{null}(\cdot)$  denotes an orthonormal basis for the null space of a matrix.

**Phase III:** Data transmission phase.

*Remark:* Let  $\mathbf{U}^{[j]}$  be the transmit precoding matrix at  $j$ -th RX, and let  $\mathbf{V}^{[j]}$  be the receive beamforming matrix at  $j$ -th TX. By using the reciprocity of alignment [4], IA is also achieved for the reverse transmission.

### B. Extension to Multiple Antenna Scenario

Based on the close-form solution in Appendix IV in [3], our scheme can be extended to multiple symmetric antenna scenario in the 3-user MIMO IC without symbol extension in time or frequency. The process is also shown in Fig. 2.

We assume each node is equipped with even number of antennas (odd antenna configuration for the 3-user MIMO IC can be deduced based on Appendix V in [3]). The process is also divided into 3 phases.

**Phase I:** Channel estimation phase.

**Phase II:** LOIA algorithm phase.

1) At RX1, RX2, and RX3, let

$$\mathbf{P}_1 = (\mathbf{H}^{[12]})^{-1} \mathbf{H}^{[13]},$$

$$\mathbf{P}_2 = (\mathbf{H}^{[23]})^{-1} \mathbf{H}^{[21]},$$

$$\mathbf{P}_3 = (\mathbf{H}^{[31]})^{-1} \mathbf{H}^{[32]},$$

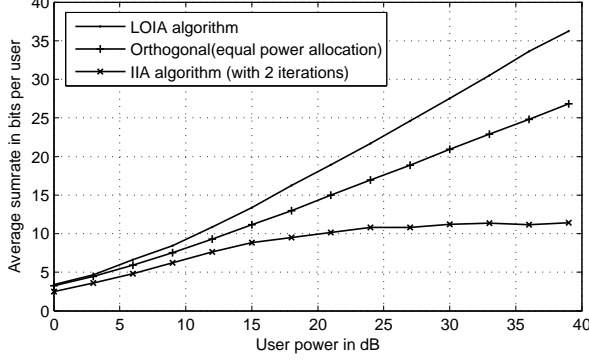


Fig. 3. Comparison of the LOIA algorithm with the IIA algorithm (algorithm 1 in [4]) for the 3-user MIMO IC in the overhead-limited scenario, where each node is equipped with 2 antennas and the number of iterations of the LOIA algorithm and the IIA algorithms equals 2.

respectively.

2) RXs send some training sequences along the vectors in  $\mathbf{P}_1$ ,  $\mathbf{P}_2$ , and  $\mathbf{P}_3$  respectively. Then  $\mathbf{P}_1$ ,  $\mathbf{P}_2$ , and  $\mathbf{P}_3$  can be estimated in all TXs.

3) Let  $\mathbf{T} = \mathbf{P}_3\mathbf{P}_1\mathbf{P}_2$ , and let

$$\mathbf{T}\mathbf{V}^{[1]} = \mathbf{V}^{[1]} \quad (7)$$

at TX1. Then choose

$$\mathbf{V}^{[1]} = [e_1, \dots, e_{M/2}]$$

at TX1, where  $e_1, e_2, \dots, e_M$  are the  $M$  eigenvectors of  $\mathbf{T}$ . At TX2 and TX3, they can calculate  $\mathbf{V}^{[1]}$  as above, and let

$$\mathbf{V}^{[2]} = \mathbf{P}_3^{-1}\mathbf{V}^{[1]}, \quad (8)$$

$$\mathbf{V}^{[3]} = \mathbf{P}_2\mathbf{V}^{[1]}, \quad (9)$$

respectively. It is shown in [3] that (7)-(9) are equivalent to (10)-(12).

$$\text{span}(\mathbf{H}^{[12]}\mathbf{V}^{[2]}) = \text{span}(\mathbf{H}^{[13]}\mathbf{V}^{[3]}), \quad (10)$$

$$\mathbf{H}^{[21]}\mathbf{V}^{[1]} = \mathbf{H}^{[23]}\mathbf{V}^{[3]}, \quad (11)$$

$$\mathbf{H}^{[31]}\mathbf{V}^{[1]} = \mathbf{H}^{[32]}\mathbf{V}^{[2]}, \quad (12)$$

where  $\text{span}(\cdot)$  denotes the vector space that spanned by the columns of a matrix. Then IA is achieved at all RXs immediately without other iteration.

4) TXs send some training sequences along vectors in  $\mathbf{V}^{[j]}$ ,  $j = 1, 2, 3$ . RXs estimate  $\mathbf{V}^{[j]}$  and let

$$\mathbf{U}^{[1]} = \text{null}(\mathbf{H}^{[12]}\mathbf{V}^{[2]}),$$

$$\mathbf{U}^{[2]} = \text{null}(\mathbf{H}^{[21]}\mathbf{V}^{[1]}),$$

$$\mathbf{U}^{[3]} = \text{null}(\mathbf{H}^{[31]}\mathbf{V}^{[1]}),$$

at RX1, RX2, and RX3 respectively. By using the reciprocity of the channel, IA is achieved at all TXs without other iteration.

**Phase III:** Data transmission phase.

We can see that GCSI is achieved through two training phases. Then precoding matrices and receive beamforming matrices can be calculated directly.

## C. Discussions

In the  $K$ -user MIMO IC, When  $K > 3$ , our scheme can be extended to this scenario whenever there has a close-form solution with symmetric antenna configuration [6]-[8]. We leave it for future work for asymmetric antenna configuration in the  $K$ -user MIMO IC.

## IV. SIMULATION RESULTS

The comparison of the IIA algorithm with the LOIA algorithm is presented in Fig. 3 for the 3-user MIMO IC, where each node is equipped with 2 antennas. We consider 1 stream allocation per user-pair for the IIA algorithm and the LOIA algorithm, and the number of iterations of the algorithms equals two. All channel coefficients are assumed independent and identically distributed (i.i.d.) zero mean unit variance circularly symmetric complex Gaussian. Orthogonal scheme is also plotted for comparison, where the sum rate is calculated assuming equal time sharing for the users, and with power  $3P$  per node.

From the simulation results we can see that the IIA algorithm is strictly suboptimal compared with our LOIA algorithm in the overhead-limited scenario. The achievable rate of the IIA algorithm even lower than orthogonal scheme in this scenario.

## V. CONCLUSION

Based on the closed-form IA solutions in [3], a LOIA algorithm is proposed in this letter for the  $K$ -user SISO interference channel, and extension to multiple antenna scenario is also considered. Compared with the IIA algorithm, the overhead of our LOIA scheme is greatly reduced.

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